Note that choice E) NOTA stands for "None of the above" answers is correct

For Questions 1-2, consider the following sequence: 1, 3, 7, 13, 21, 31, 43, ___, ….

1. Find the next term in the sequence, where $a_n = a_{n-1} + 2n$ and $a_0 = 1$.

A) 56   B) 57   C) 58   D) 59   E) NOTA

2. Find the sum of the squares of the roots for the quadratic expression in terms of n that defines the a_n term in the sequence. (a_0=1, a_1=3, a_2=7, etc.)

A) -1   B) 0   C) 1   D) 2   E) NOTA

3. With like terms combined, how many terms are in the expansion of $(2a+b+c+19d)^{12}$?

A) 455   B) 960   C) 1365   D) 1820   E) NOTA

4. Evaluate:

$$\sqrt{182 + \sqrt{182 + \sqrt{182 + \sqrt{182 + \ldots}}}}$$

A) -14   B) -13   C) 13   D) 14   E) NOTA

5. Find the next term: 5, 7, 11, 12, 17, ___, …. (hint: spell out each term—what letter is shared by only those terms up through 17?).

A) 19   B) 23   C) 25   D) 27   E) NOTA

6. There exists a positive integer P such that

$$\sum_{n=1}^{25} n^3 = p^2$$

Find the sum of the digits of P.

A) 7   B) 8   C) 9   D) 10   E) NOTA

7. Find the common ratio of a geometric sequence that has a first term of -2 and a fourth term of -686.

A) $\sqrt{7}$ B) $-\sqrt{7}$ C) 7   D) -7 E) NOTA

8. Find the sum of the first 10 terms of the infinite series $a_n = 1+3n$, such that $a_0 = 1$, $a_1 = 4$, etc.

   A) 117   B) 145   C) 176   D) 210   E) NOTA

9. Find the 2018th term of the non-decreasing sequence: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4… , where each positive integer appears its own number of times.

   A) 61   B) 62   C) 63   D) 64   E) NOTA

10. A geometric sequence has a seventh term of 12 and a common ratio of $-1+i$, where $i = \sqrt{-1}$. Find the twenty-third term.

   A) $-1536 + 1536i$ B) $-1536 - 1536i$ C) 3072 D) -3072
                                                                                          E) NOTA

11. Adam has 100 closed doors labeled 1 through 100. He gets 100 of his classmates and numbers them off such that no two people share the same number and all whole numbers from 1 to 100 are accounted for. Then, every one of his classmates must go and reverse the orientation of all doors that are multiples of their assigned number (If a door was closed, it would be opened, and vice versa), starting with classmate 1 and ending with classmate 100. For example, number one would open all of the doors, then number two would close all even numbered doors, number three would change the orientation of multiples of 3, etc. How many doors are open after all 100 classmates have had their turn?

   A) 10   B) 25   C) 75   D) 90   E) NOTA

12. Jerry and Beth are playing a game with two standard fair six-sided dice. If Jerry rolls two prime numbers, he wins. If Beth rolls two composite numbers, she wins. The two take turns rolling both dice until one of them wins. What is the probability that Beth wins if Jerry rolls first?

   A) 1/4   B) 1/8   C) 8/11   D) 1/9   E) NOTA

13. Evaluate the sum:
$$\sum_{x=1}^{\infty} \frac{3}{x^2 + 11x + 28}$$

A) $\dfrac{107}{210}$ B) $\dfrac{11}{30}$ C) $\dfrac{533}{840}$ D) $\dfrac{8}{15}$ E) NOTA

14. Find the sum of the lowest 17 positive even square numbers.

   A) 7140   B) 5984   C) 8436   D) 9880   E) NOTA

15. A sequence is defined as $a_n = n^{a_{n-1}} - 2^{a_{n-2}}$ for $n \geq 3$. If $a_1 = a_2 = 1$, find the value of

$$\sum_{m=2}^{5} (a_m + 2^m)$$

A) 85   B) 87   C) 89   D) 91   E) NOTA

16. Given a convex enneagon has integer angle measures that follow an arithmetic sequence with a common difference that is an integer, what is the smallest angle possible in the polygon?

   A) 100   B) 104   C) 110   D) 112   E) NOTA

17. Jimmy Jim makes a number using the first n odd numbers, e.g., 1357911. How many digits would Jimmy Jim's number have if he uses the first 321 odd numbers.

   A) 908   B) 911   C) 917   D) 920   E) NOTA

18. Compute the following sum where the numerators are the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, . . . and the denominators are the successive powers of 6. Your answer should be a rational number.

$$S = \frac{1}{6^0} + \frac{1}{6^1} + \frac{2}{6^2} + \frac{3}{6^3} + \frac{5}{6^4} + \frac{8}{6^5} + \frac{13}{6^6} + \ldots$$

A) $\dfrac{7}{29}$ B) $\dfrac{6}{5}$ C) $\dfrac{1296}{29}$ D) $\dfrac{36}{29}$

E) NOTA

19. Suppose there exists a function f(m) such that
$$f(m+1) = m(-1)^m - 2f(m)$$
$$f(1) = f(2019)$$

Find
$$f(1) + f(2) + f(3) + f(4) + \ldots + f(2018).$$

A) $\dfrac{1009}{2}$ B) 1009   C) $\dfrac{1009}{3}$ D) $\dfrac{2018}{3}$ E) NOTA

20. $4.19\overline{2}$ (with the digit 2 repeating) can be expressed as $\dfrac{a}{b}$ with a and b being relatively prime positive integers. Find $a + b$.

   A) 3773   B) 4673   C) 5142   D) 5223   E) NOTA

21. John slices his circular pizza into n pieces using 10 cuts, while Murphy wants to slice his circular pizza into m pieces with 11 cuts. Find the maximum value of $n + m$.

   A) 123   B) 102   C) 146   D) 113   E) NOTA

22. Two arithmetic sequences contain the numbers 1, 7, 13… 1003 and 3, 8, 13… 1003 respectively. How many numbers are common to both sequences?

   A) 33   B) 34   C) 66   D) 67   E) NOTA

23. Find the value of

$$\sum_{m=0}^{\infty} \frac{8m - 3}{2^m}$$

   A) 6   B) 16   C)14   D) 8   E) NOTA

24. Find the value of x given:

$$x = 4 + \cfrac{1}{4 + \cfrac{1}{4+\dots}} - 1$$

   A) $1 + \sqrt{5}$ B) $2 + \sqrt{5}$ C) $4+\sqrt{3}$ D) $3 + \sqrt{13}$ E) NOTA

25. How many distinct geometric sequences of integers exist so that the first term is 1 and the last term is 1024?

   A) 1   B) 4   C) 5   D) 6   E) NOTA

26. Bup drops a meatball from a height of 20 ft. The rubbery meatball bounces in a way that the ratio of the height of one peak (beginning with the initial drop of 20 ft) to the height of the next peak forms a harmonic sequence of $\dfrac{1}{n+1}$ where n is the number of times the ball has already bounced. For example, the second peak would have a height of 10ft where n=1. What is the meatball's total vertical distance travelled, in feet, at the moment of the 6th bounce?

   A) $\dfrac{1810}{63}$ B) $\dfrac{2165}{63}$ C) $\dfrac{3070}{63}$ D) $\dfrac{4330}{63}$ E) NOTA

27. Rick created an infinite number of regular octagons by constructing each subsequent octagon by connecting the midpoints of the previous octagon. He begins with an octagon of side length 2 units. Find the sum of the enclosed areas of each octagon.

A) $96 + 64\sqrt{2}$　　B) $64 + 48\sqrt{2}$　　C) $24 + 16\sqrt{2}$　　D) $96 + 48\sqrt{2}$　　E) NOTA

28. Evaluate

$$\sum_{n=18}^{30} (n^3 - 51n^2 + 867n - 4913)$$

A) 8281　　　　B) 10215　　　　C) 6094　　　　D) 6785　　　　E) NOTA

29. The elevation of the peak of Pikes Peak is 14115 feet. Morty is at the bottom of the mountain (elevation of 0 feet) and wants to climb to the peak. He starts on Monday and hikes 200 feet vertically. On each successive day, Morty manages to hike 50 more feet vertically than the previous day. On which day of the week does Morty reach the peak?

A) Monday　　　　B) Tuesday　　　　C) Thursday　　　　D) Friday　　　　E) NOTA

30. Find the sum of the 20th triangular number and 18th hexagonal number.

A) 951　　　　B) 893　　　　C) 840　　　　D) 990　　　　E) NOTA